Statistical modeling and simulation of delay-Doppler maps in the time-varying regime and comparison with real airborne data

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Motivation and Objectives

- The analysis of temporal dynamics in GNSS signals scattered off the sea surface is a relevant and challenging problem.
- The time-varying factors ruling the GPS scattered signal can be grouped in two main categories: variation of the system geometry and motion of the sea surface.
- An accurate physical/statistical model of the scattered signal should consider time-variations.
- The aim of this work is to introduce time evolving sea state and platforms motion in the GNSS-R stochastic model [1], and to show its consistency with real data.

[1] G. Giangregorio, et al., "Stochastic Modeling and Simulation of Delay-Doppler Maps in GNSS-R Over the Ocean," IEEE Transactions on Geoscience and Remote Sensing, April 2016

Time-varing stochastic model

The received signal is the sum of scattering from single delay Doppler cells:

$$u(t) = \sum_m \frac{\mathcal{D}_m \mathcal{R}_m}{2R_m R_{0m}} e^{ik(R_{0m}(t)+R_m(t))} a(t-\tau_m) \sum_{j=1}^{N_m(t)} A_j(t) e^{i\phi_j(t)}$$

The random sum, known as a *random walk* in two dimensions, converges to a Compound-Gaussian process

$$\lim_{\overline{N}_m \to \infty} \frac{1}{\sqrt{\overline{N}_m}} \sum_{i=1}^{N_m(t)} A_i(t) e^{i\phi_i(t)} \stackrel{d}{=} Z_m(t) = \sqrt{S_m(t)} G_m(t).$$

- $N_m(t)$ Birth-Death-Immigration process with birth-rate λ, death-rate μ and immigration-rate ν;
- $S_m(t)$ Gamma-distributed process $\Gamma(\alpha, \alpha)$ with $\alpha = \nu/\lambda$ and $\lambda < \mu$, with unit mean: it accounts for the distribution of the number of specular points $N_m(t)$;
- $G_m(t)$ complex Gaussian process, the speckle component, with variance $\sigma_m^2 = \frac{\mathcal{A}_m}{2} \frac{q_m^4}{q_{z\,m}} P\left(-\frac{\vec{q}_{\perp m}}{q_{z\,m}}\right)$: it accounts for the randomness of scattering from the specular points.

Time-varying characterization of the correlator output

The signal at the correlator output $Y(t,\tau,f)$ is

$$T\sum_m \frac{\mathcal{D}_m\mathcal{R}_m}{2R_mR_{0m}} e^{i2\pi(f-f_{Dm})t} e^{ik(R_m+R_{0m})}\sqrt{S_m(t)}G_m(t)\chi(\tau-\tau_m, f-f_{Dm})$$

Its Auto Correlation Function $R_Y(\tilde{t},\tau,f)$ is

$$T^2\sum_m \frac{\mathcal{D}_m^2|\mathcal{R}_m|^2|\chi(\Delta\tau_m,\Delta f_m)|^2}{4R_m^2R_{0m}^2}\mathcal{A}_m\frac{q_m^4}{q_{z_m}^4}P\left(-\frac{\vec{q}_{\perp m}}{q_{z_m}}\right)e^{i2\pi\Delta f_m\tilde{t}}e^{-\frac{U}{3\lambda}|\tilde{t}|}$$

The correlation time of the Compound Gaussian process has been assumed as $\frac{3\lambda}{U}$, where U is the wind speed and λ is GNSS signal wavelength[2].

[2] E. Valencia, et al., "Experimental Determination of the Sea Correlation Time Using GNSS-R Coherent Data," IEEE Geoscience and Remote Sensing Letters, Oct. 2010

Generation of the istantaneous DDM

The time evolution of the delay-Doppler map is computed as the 2-D discrete-time convolution

$$Y(t,\tau,f) = \sum_{i\in I}\sum_{j\in J}\Theta(t,\tau_i,f_j)\chi(\tau-\tau_i,f-f_j)e^{i2\pi(f-f_j)t}$$

where the time-varying scattering function is:

$$\Theta(t,\tau_i,f_j) = T\left[W_1(\tau_i,f_j)Z_1(t,\tau_i,f_j) + W_2(\tau_i,f_j)Z_2(t,\tau_i,f_j)\right]$$

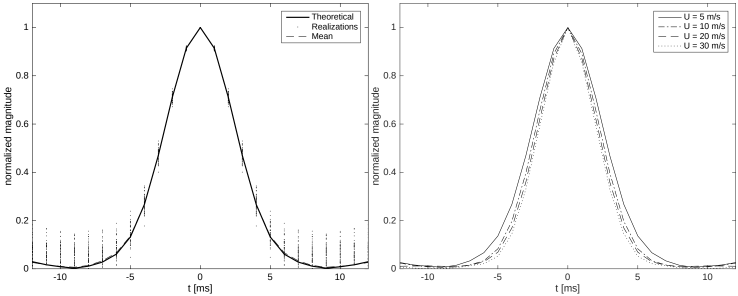
The function $W_n(\tau_i,f_j)$ includes the system geometry while $Z_n(t,\tau_i,f_j)$ is the correlated compound-Gaussian process [3].

[3] $n = 1,2$ indicates the ambiguity, i.e. the two spatial components in the (τ_i,f_j) delay Doppler cell

Numerical simulation for airborne configuration

Simulated and theoretical normalized magnitude acf of the signal from the nominal specular point.

Receiver height $h = 3200\ m$
Receiver velocity $v_r = 119.4\ m/s$



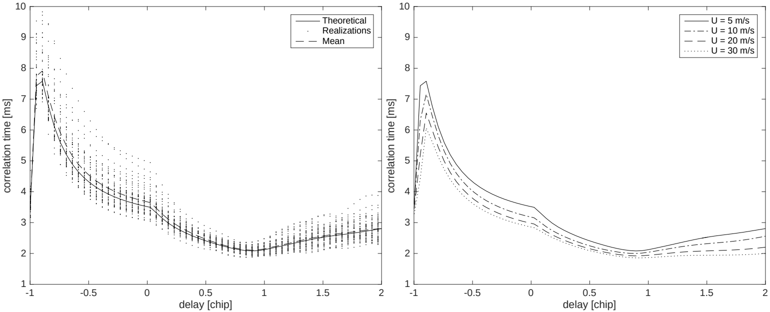
Gaussian trend, Non-linear least squares Gaussian fitting.

Estimated correlation time $\hat{t}_{cor} = 3.70$ ms

Theoretical correlation time $\tilde{t}_{cor} = 3.53$ ms.

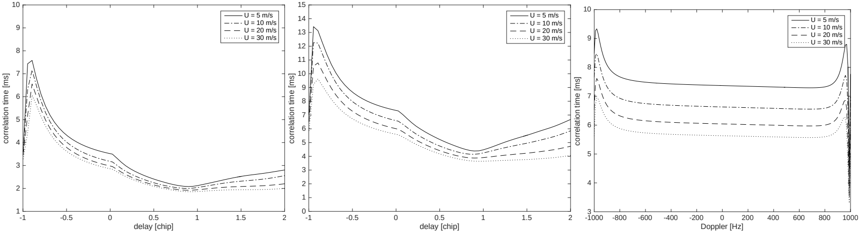
...Numerical simulation for airborne configuration

Correlation time versus range delay at zero-Doppler.



Receiver height $h = 3200\ m$
Receiver magnitude velocity $v_r = 119.4\ m/s$.

...Numerical simulation for airborne configuration



$h = 3200\ m$
$v_r = 119.4\ m/s$

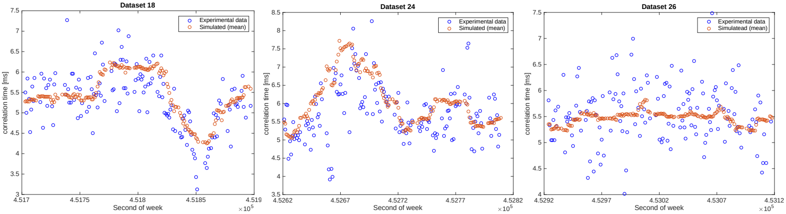
$h = 3000\ m$
$v_r = 55\ m/s$

Correlation time for different platform velocity and wind speed

	$U = 9$ m/s	$U = 15$ m/s	$U = 30$ m/s
v_r [m/s]	$\tilde{t}_{cor}$ [ms]	$\tilde{t}_{cor}$ [ms]	$\tilde{t}_{cor}$ [ms]
55	6.7205	6.2946	5.634
85	4.3796	4.141	3.8145
119	3.2153	3.0508	2.8458

Comparisons with GOLD-RTR real data

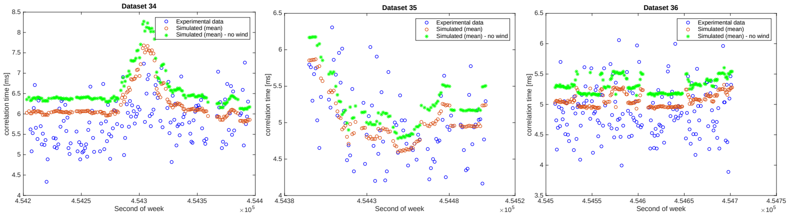
Data collected by GOLD-RTR during ESA aicraft mission in the Gulf of Finland on the 11th Nov 2011



Correlation time evaluated every second, for different datasets, in comparison with the model

Comparisons with GOLD-RTR real data

Theoretical correlation times with and without the correlation term due to the Compound Gaussian (CG) process.



If the sea surface motion is not considered [4]*, the correlation time is slightly overestimated*

[4] H. You, et al., "Stochastic voltage model and experimental measurement of ocean-scattered GPS signal statistics," IEEE Transactions on Geoscience and Remote Sensing, Oct 2004.

Mean Square Error (MSE)

MSE between the correlation time of the real data and of the model, obtained with and without the correlation due to the compound Gaussian process

	Dataset34	Dataset35	Dataset36	Dataset38
MSE (with CG corr)	0.2278	0.2235	0.5187	0.3474
MSE (without CG corr)	0.3012	0.3798	0.9669	0.4976

Conclusions and future work

1. A time-varying stochastic model has been proposed where the signal correlation due to both the geometrical variations and the sea surface motion is included.
2. Theoretical and simulated results are in agreement, showing that the correlation time depends on the platform velocity and also on the wind speed, at least in airborne configuration.
3. Results have been compared against real data. The correlation of the stochastic process provides a more accurate model for the scattered signal with respect to the model that considers only the decorrelation due to the geometry.
4. The knowledge of the signal correlation may be exploited in the design of an improved processing at the receiver, with the aim of increasing the SNR of the delay Doppler map.